A Simple Method to Calculate Mode Components of Strain Energy Release Rate of Free-Edge Delaminations in Composite Laminates

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A simple method, which calculates the mode components of the strain energy release rate of free-edge delaminations in the laminates, is proposed. The interlaminar stresses are evaluated as an interface moment and interface shear forces that are obtained from the equilibrium equations at the interface between the adjacent layers. Deformation of an edge-delaminated laminate is calculated by using a generalized quasi-three dimensional classical lamination theory developed by the author. The analysis provides closed-form expressions for the three components of the strain energy release rate. The analyses are performed on $[+30/-30/90]_s$ laminates subjected to uniaxial extension, with free-edge delaminations located symmetrically and asymmetrically with respect to the laminate midplane. Comparison of the results with a finite element solution using the virtual crack closure technique shows good agreement. The simple nature of this method makes it suitable for primary design analysis for the delaminations of composite laminates.

Key Words: Generalized Quasi-Three Dimensional Classical Lamination Theory (GQ3D-CLT), Free-Edge Delamination, Stranin Energy Release Rate, Composite Laminates

1. Introduction

Delaminations along the free-edges of composite laminates under uniform axial strain as shown in Fig. 1 have been observed during testing and in service. The free-edge delaminations induce redistribution of the stresses in the plies of the laminate, and therefore, usually result in a reduction of stiffness and strength of the laminate. A number of experimental and analytical investigations have been directed toward well understanding of free-edge delamination mechanisms.

Pipes and Pagano (1970) proposed a quasi-three dimensional (Q3D) analysis for a symmetric laminate under uniform axial extension. They showed the interlaminar stress distribution by the finite difference technique based on the Q3D analysis, and pointed out that free-edge delaminations are caused by the interlaminar stresses which arise in the vicinity of the free-edge. Rybicki, Schmueser and Fox (1977) and Wang and Crossman (1980) evaluated the strain energy release rate (SERR) by using a finite element method (FEM) with the virtual crack closure technique. These previous works have shown that the delamination onset and growth can be characterized quantitatively by the SERR. In order to estimate the SERR, O'Brien (1982) developed a simple expression based on the classical laminated plate theory (CLT) and the rule of mixtures.

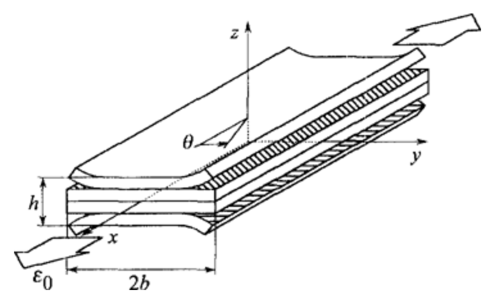

Fig. 1 A laminate with free-edge delaminations and coordinate system.

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Aoki and Kondo(1989) proposed a method based on the J-integral law in combination with the CLT for calculating the Mode-I component of SERR. Armanios and Rehfield(1989) used a sublaminate analysis with a shear deformable plate theory to determine the individual mode components of SERR. Schapery and Davidson (1990) proposed a crack tip element approach, which is a method that combined a sublaminate analysis based on the CLT with FEM, for determining mode ratio of SERR.

The objective of the present work is to develop a simple method for determining the individual mode components of SERR. The method is based on the GQ3D-CLT. The analysis provides closed-form expressions for the Mode-I, Mode-II, and Mode-III component of SERR.

2. Formulation

2.1 Evaluation of interlaminar stresses

Classical laminated plate theory(CLT) assumes that the state of stress within each lamina of a multidirectional laminate is planar. This assumption is accurate for inner regions away from laminate geometric discontinuities, such as free-edge. In the vicinity of the free-edge a boundary exist where the state of stress is three dimensional. The boundary width is approximately equal to the overall laminate thickness (Pipes and Pagano, 1970). The stresses which arise at the interface between adjacent layers in the boundary are called interlaminar stresses. A lot of studies have been conducted to analyze the distribution of the interlaminar stresses. In this paper, we evaluate indirectly the interlaminar stresses as stress resultants instead of calculating the interlaminar stress distribution. CLT gives the in-plane stresses, σ_y and τ_{xy} shown in Fig. 2.

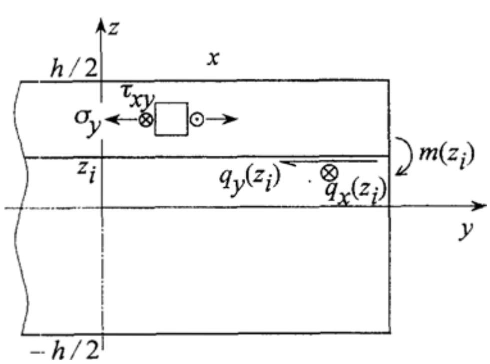


Fig. 2 Definition of interface moment and interface shear forces.

These in-plane stresses vanish at the free-edge of the laminate. By using equilibrium equations of stress resultants at the interface $z=z_i$ in the boundary, we define an interface moment (Halpin, 1984) $m(z_i)$ and interface shear forces $q_y(z_i)$ and $q_x(z_i)$ as follows :

$$
\begin{aligned}
m(z_i) &= \int_{z_i}^{h/2} \sigma_y(z)\,(z-z_i)\,dz \\
q_y(z_i) &= -\int_{z_i}^{h/2} \sigma_y(z)\,dz \\
q_x(z_i) &= -\int_{z_i}^{h/2} \tau_{xy}(z)\,dz
\end{aligned} \tag{1}
$$

The interface moment $m(z_i)$ is reacted by the interlaminar normal stress component σ_z. The distribution of the interlaminar normal stress must, therefore, result in zero vertical force vector while producing a moment equal in magnitude to that given by the first equation of Eqs. (1). When the interface moment is positive, the interlaminar normal stress σ_z is tensile in the boundary near the free-edge. It means that a peeling stress occurs at the interface in the free-edge boundary and delamination crack with the opening mode (Mode-Ⅰ) may be generated.

The interface shear forces, $q_y(z_i)$ and $q_x(z_i)$, are reacted by the interlaminar shear stress components τ_{yz} and τ_{xz}, respectively. When the interface shear force is not zero, the interlaminar shear stress occurs at the interface in the free-edge boundary. In the case of $q_y(z_i) \neq 0$, delamination crack with the in-plane shear mode (Mode-Ⅱ) may be generated. In the case of $q_x(z_i) \neq 0$,

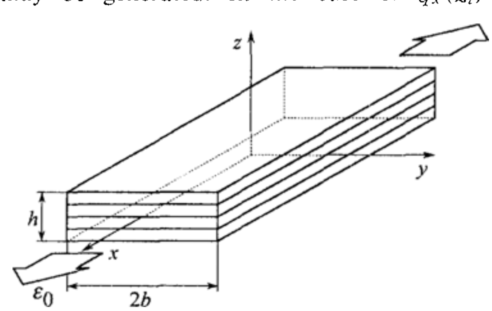


Fig. 3 Quasi-three dimensional problem.

delamination crack with the anti-plane shear mode (Mode-Ⅲ) may be generated.

2.2 Generalized quasi-three dimensional classical laminated plate theory

Pipes and Pagano (1970) proposed the following quasi-three dimensional (Q3D) displacement field equations for symmetric laminates under uniform axial strain as shown in Fig. 3.

$$\begin{aligned} u(x, y, z) &= U(y, z) + \varepsilon_0 x \\ v(x, y, z) &= V(y, z) \\ w(x, y, z) &= W(y, z) \end{aligned} \tag{2}$$

where ε_0 is uniform axial strain. These Q3D displacement field equations, however, are not applicable to asymmetric laminates. Hence we derived the following displacement field equations for asymmetric laminates.

$$\begin{aligned} u(x, y, z) &= U(y, z) + (C_1 + C_2 z)x \\ v(x, y, z) &= V(y, z) + C_3 z x \\ w(x, y, z) &= W(y, z) - (\tfrac{1}{2}C_2 x + C_3 y)x \end{aligned} \tag{3}$$

We called Eqs. (3) as generalized quasi-three dimensional (GQ3D) displacement field equations. While the parameter C_1 is equivalent to ε_0, which describes uniform axial strain of a laminate, the GQ3D equations include additional parameters C_2, C_3, and C_4. C_2 and C_4 represent bending deformations of a laminate, and C_3 represents twisting deformation of a laminate.

We derive a GQ3D-CLT by applying the GQ3D displacement field Eqs. (3) to CLT. CLT gives the in-plane strain components of a laminate as follows :

$$\varepsilon_x = \varepsilon_x^0 + z\chi_x,\ \varepsilon_y = \varepsilon_y^0 + z\chi_y,\ \gamma_{xy} = \gamma_{xy}^0 + z\chi_{xy} \tag{4}$$

where

$$\begin{aligned} &\varepsilon_x^0 = \frac{\partial u^0}{\partial x},\ \varepsilon_y^0 = \frac{\partial v^0}{\partial y},\ \gamma_{xy}^0 = \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x}, \\ &\chi_x = -\frac{\partial^2 w^0}{\partial x^2},\ \chi_y = -\frac{\partial^2 w^0}{\partial y^0},\ \chi_{xy} = -2\frac{\partial^2 w^0}{\partial x \partial y} \end{aligned} \tag{5}$$

u^0, v^0, and w^0 are displacements at the midplane of the laminate. On the other hand, we get the following x-direction strain by the use of the GQ3D displacement field Eqs. (3).

$$\varepsilon_x = \frac{\partial u}{\partial x} = C_1 + C_4 y + C_2 z \tag{6}$$

Comparing Eq. (6) with Eq. (4), we obtain the following relations.

$$C_1 + C_4 y = \varepsilon_x^0,\ C_2 = \chi_x \tag{7}$$

We consider the laminate under uniform axial ε_0. Therefore,

$$C_1 = \varepsilon_0 \tag{8}$$

Introducing the curvature ω_x with respect to the in-plane bending moment, we get the following equation.

$$C_4 = \omega_x \tag{9}$$

Substituting the GQ3D displacement field Eqs. (3) to the last equation of Eqs. (5), we obtain

$$C_3 = \chi_{xy} \tag{10}$$

We can find that the GQ3D displacement field Eqs. (3) are applicable to analyze a laminate under the bending deformations, χ_x and ω_x, and the twisting deformation χ_{xy}, in addition to axial extension ε_0, as shown in Fig. 4.

CLT defines the resultant forces and moments as follows :

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{z_1}^{z_2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz,\ \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{z_1}^{z_2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} z dz \tag{11}$$

where z_1 and z_2 are coordinates of the yz section. GQ3D-CLT, however, treats the resultant forces

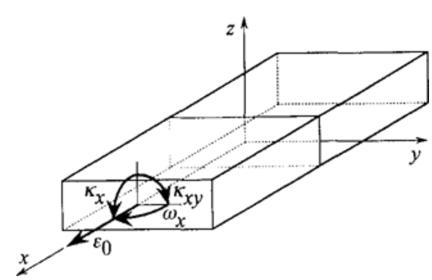


Fig. 4 Generalized quasi-three dimensional problem.

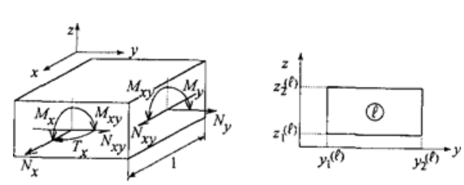


Fig. 5 Resultant forces and moments acting on a laminate with finite width.

and moments shown in Fig. 5. CLT does not include a resultant moment T_x corresponding to the curvature ω_x. In addition, CLT defines Eqs. (11) as forces and moments per unit width of the yz section because of the infinite plate assumption of CLT. Hence we employ the following definitions for the resultant forces and moments acting on the laminate with finite width (y_2-y_1) as shown in Fig. 5.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{z_1}^{z_2}\int_{y_1}^{y_2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dydz, \quad \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{z_1}^{z_2}\int_{y_1}^{y_2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} zdydz \tag{12}$$

$$T_x = \int_{z_1}^{z_2}\int_{y_1}^{y_2} \sigma_x y dydz$$

The stress-strain relations for the k-th layer of a multilayered laminate can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \tilde{Q}_{11} & \tilde{Q}_{12} & \tilde{Q}_{16} \\ \tilde{Q}_{12} & \tilde{Q}_{22} & \tilde{Q}_{26} \\ \tilde{Q}_{16} & \tilde{Q}_{26} & \tilde{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{13}$$

$$= \begin{bmatrix} \tilde{Q}_{11} & \tilde{Q}_{12} & \tilde{Q}_{16} \\ \tilde{Q}_{12} & \tilde{Q}_{22} & \tilde{Q}_{26} \\ \tilde{Q}_{16} & \tilde{Q}_{26} & \tilde{Q}_{66} \end{bmatrix}_k \left(\begin{Bmatrix} \varepsilon_0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \chi_x \\ \chi_y \\ \chi_{xy} \end{Bmatrix} + y \begin{Bmatrix} \omega_x \\ 0 \\ 0 \end{Bmatrix} \right)$$

Substituting Eq. (13) to Eqs. (12), we can derive the following constitutive equation for the GQ3D-CLT.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ T_x \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & C_{11} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & C_{12} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & C_{16} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & E_{11} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & E_{12} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & E_{16} \\ C_{11} & C_{12} & C_{16} & E_{11} & E_{12} & E_{16} & F_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \chi_x \\ \chi_y \\ \chi_{xy} \\ \omega_x \end{Bmatrix} \tag{14}$$

where

$$[A_{ij}, B_{ij}, D_{ij}] = (y_2-y_1)\sum_{k=1}^{N} (\tilde{Q}_{ij})_k \left[(z_k - z_{k-1}), \frac{1}{2}(z_k^2 - z_{k-1}^2), \frac{1}{3}(z_k^3 - z_{k-1}^3) \right]$$

$$[C_{ij}, E_{ij}] = \frac{1}{2}(y_2^2 - y_1^2)\sum_{k=1}^{N} (\tilde{Q}_{ij})_k \left[(z_k - z_{k-1}), \frac{1}{2}(z_k^2 - z_{k-1}^2) \right]$$

$$F_{11} = \frac{1}{3}(y_2^3 - y_1^3)\sum_{k=1}^{N} (\tilde{Q}_{ij})_k (z_k - z_{k-1}) \tag{15}$$

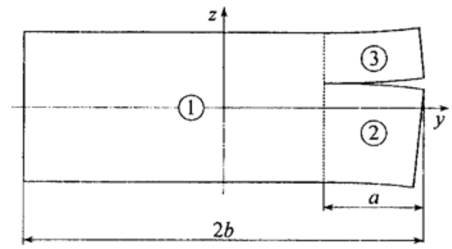


Fig. 6 Sublaminate description.

2.3 Analysis of a laminate with a free-edge delaminations by GQ3D-CLT

We consider a laminate with a free-edge delamination under uniform axial strain. The laminate is divided into three sublaminates as shown in Fig. 6.

The delamination length is denoted by a. Sublaminate 2 and 3 represent the groups of plies below and above interface along which delamination occurs, respectively. Because the forces, N_y and N_{xy}, and the moment, M_y, are not reacted on the laminate under uniform axial strain, the following conditions should be satisfied for each sublaminate.

$$N_y^{(\ell)} = N_{xy}^{(\ell)} = M_y^{(\ell)} = 0 \tag{16}$$

where superscript (ℓ) denotes the ℓ-th sublaminate. Hence we get the following reduced constitutive equations for the ℓ-th sublaminate.

$$\tilde{N}^{(\ell)} = \hat{H}^{(\ell)}\,\tilde{C}^{(\ell)} \tag{17}$$

where

$$\tilde{N}^{(\ell)} = \begin{Bmatrix} N_x \\ M_x \\ M_{xy} \\ T_x \end{Bmatrix}^{(\ell)}, \quad \hat{H}^{(\ell)} = \begin{bmatrix} A'_{11} & B'_{11} & B'_{16} & C'_{11} \\ B'_{11} & D'_{11} & D'_{16} & E'_{11} \\ B'_{16} & D'_{16} & D'_{66} & E'_{16} \\ C'_{11} & E'_{11} & E'_{16} & F'_{11} \end{bmatrix}^{(\ell)},$$

$$\tilde{C}^{(\ell)} = \begin{Bmatrix} \varepsilon_0 \\ \chi_x \\ \chi_{xy} \\ \omega_x \end{Bmatrix}^{(\ell)} \tag{18}$$

Vectors $\tilde{C}^{(1)}$, $\tilde{C}^{(2)}$, and $\tilde{C}^{(3)}$, which describe deformation of each sublaminate, can be proved to be equal since the displacements of each sublaminate are the same at the interface between the adjacent sublaminates. In addition, it is clear that the force and the moments reacted on the whole laminate are equal to the sums of the corresponding force and moments on each sublaminate. Hence we obtain the following reduced constitutive equations for the whole laminate.

$$\tilde{N}^{LAM}=\tilde{H}^{LAM}\ \tilde{C}^{LAM} \tag{19}$$

where

$$\begin{aligned}\tilde{N}^{LAM}&=\tilde{N}^{(1)}+\tilde{N}^{(2)}+\tilde{N}^{(3)}\\ \tilde{H}^{LAM}&=\tilde{H}^{(1)}+\tilde{H}^{(2)}+\tilde{H}^{(3)}\\ \tilde{C}^{LAM}&=\tilde{C}^{(1)}=\tilde{C}^{(2)}=\tilde{C}^{(3)}\end{aligned} \tag{20}$$

Superscript LAM denotes the whole laminate.

We can obtain deformation of each sublaminate by solving the constitutive Eq. (19). Substituting $\tilde{C}^{(1)}$, $\tilde{C}^{(2)}$, and $\tilde{C}^{(3)}$ to Eq. (17) and using Eq. (16), we can obtain strains and curvature, $\varepsilon_y^{0(\ell)}$, $\gamma_{xy}^{(\ell)}$ and $\chi_y^{(\ell)}$ of each sublaminate from Eq. (14) written in terms of the ℓ-th sublaminate.

2.4 The simple method of strain energy release rate of a free-edge delaminations

We can obtain the strain energy release per unit length of the laminate by the onset of a delamination crack of length a as follows :

$$U(a)=\frac{1}{2}(N_x^{LAM}\varepsilon_0^{LAM}+M_x^{LAM}\chi_x^{LAM}+M_{xy}^{LAM}\chi_{xy}^{LAM}+T_x^{LAM}\omega_x^{LAM}) \tag{21}$$

We obtain the following equation for the total strain energy release rate under the constant displacement condition.

$$G(a)=-\frac{U(a+\Delta a)-U(a)}{\Delta a} \tag{22}$$

where Δa is a virtual crack propagation length.

As we mentioned in section 2. 1, a peeling stress occurs at the interface $z=z_i$ in the free-edge boundary when the interface moment $m(z_i)$ is positive. If a delamination crack of length a with the opening mode (Mode-Ⅰ) is generated, we can calculate the opening angle of the crack by $[\chi_y^{(3)}-\chi_y^{(2)}]a$, where $\chi_y^{(3)}$ and $\chi_y^{(2)}$ represent the curvature of sublaminates above and below the interface $z=z_i$ along which delamination occurs, respectively. We get the Mode-Ⅰ strain energy release by the onset of the delamination crack by as follows:

$$U_{\rm I}(a)=\frac{1}{2}m(z_i)[-\chi_y^{(3)}+\chi_y^{(2)}]a \tag{23}$$

We can obtain the inplane sliding displacement of the crack by $[\varepsilon_y^{0(3)}-\varepsilon_y^{0(2)}]a$ and the anti-plane tearing displacement of the crack by $[\gamma_{xy}^{0(3)}-\gamma_{xy}^{0(2)}]a$, as well. Using the interface shear force $q_y(z_i)$ and $q_x(z_i)$, we get the Mode-Ⅱ and Mode-Ⅲ strain energy release due to the onset of the delamination crack by the following equations, respectively.

$$\begin{aligned}U_{\rm II}(a)&=\frac{1}{2}q_y(z_i)[\varepsilon_y^{0(3)}-\varepsilon_y^{0(2)}]a\\ U_{\rm III}(a)&=\frac{1}{2}q_x(z_i)[\gamma_{xy}^{0\ (3)}-\gamma_{xy}^{0\ (2)}]a\end{aligned} \tag{24}$$

We can therefore, calculate the Mode-Ⅰ, Mode-Ⅱ, and Mode-Ⅲ components of SERR for the free-edge delamination of length by the following equations, respectively.

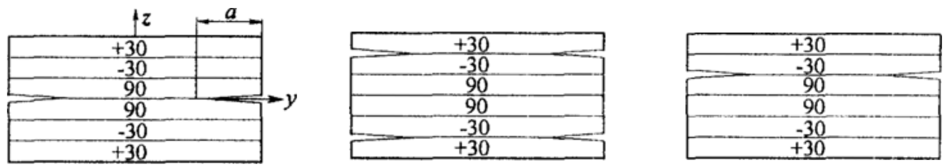
$$\begin{aligned}G_{\rm I}(a)&=U_{\rm I}(a)/a=\frac{1}{2}m(z_i)[-\chi_y^{(3)}+\chi_y^{(2)}]\\ G_{\rm II}(a)&=U_{\rm II}(a)/a=\frac{1}{2}q_y(z_i)[\varepsilon_y^{0(3)}-\varepsilon_y^{0(2)}]\\ G_{\rm III}(a)&=U_{\rm III}(a)/a=\frac{1}{2}q_x(z_i)[\gamma_{xy}^{0\ (3)}-\gamma_{xy}^{0\ (2)}]\end{aligned} \tag{25}$$

3. Numerical Examples and Discussion

The proposed method is applied to determine the mode components of SERR for free-edge delaminations in the $[+30/-30/90]_s$ composite laminate under uniform axial extension. The material property and the geometry of the laminate are given in Table 1.

Free-edge delaminations exist at the interfaces which are symmetrically located with respect to the $z=0$ plane (midplane) and the $y=0$ plane as shown in Fig. 1. Because of symmetries, only one quarter of the $x=0$ section was analyzed. We utilized a GQ3D-FEM (Uda et al., 1995) with the virtual crack closure technique to evaluate the accuracy of the present simple method. The FEM code uses eight-node quadrilateral isoparametric elements.

Most analyses to obtain the SERR for composite laminates with delamination assume that individual plies or ply groups may be modeled as



(a) Symmetric delaminations of [+30/−30/90]$_s$ laminate (b) Asymmetric delaminations of [+30/−30/90]$_s$ laminate

Fig. 7 Geometry and coordinate system of analyzed composite laminate.

Table 1 Material properties and geometry of the laminate.

$E_L = 138.6$ GPa	$E_T = 10.07$ GPa
$G_{LT} = 4.117$ GPa	$G_{TT} = 3.873$ GPa
$\nu_{LT} = 0.3200$	$\nu_{TT} = 0.3000$
b (Semi-width) $= 15$ mm	
h (Laminate thickness) $= 6h_0 = 0.84$ mm	
h_0 (Ply thickness) $= 0.14$ mm	

L denotes the fiber direction and T denotes the transverse direction.

homogeneous and orthotropic. When this assumption is adopted and the delamination is between plies at dissimilar angles, a linear elastic analysis cannot define the individual SERR components uniquely, because of a near-field oscillatory singularity. (Raju et al., 1988) Raju, Crews and Aminpour(1988) analyzed the individual SERR components by the finite element model with a thin resin layer at the delamination interface to eliminate the oscillatory singularity. They showed that the finite element model with the ' bare' interface, where the resin layer does not exist, is a very good approximation to the case with the interface resin layer, when the virtual crack propagation length Δa is either 0.25 or 0.5 of the ply thickness h_0. Hence we took $\Delta a/h_0 = 0.36$ in FEM.

We analyzed three cases, that is (1) [+30/−30/90]$_s$ laminate with delaminations at the 90/90 interface and (2) [+30/−30/90]$_s$ laminate with delaminations at the +30/−30 interfaces and (3) [+30/−30/90]$_s$ laminate with asymmetric delaminations at the −30/90 interface.

3.1 [+30/−30/90]$_s$ laminate with delaminations at the 90/90 interface

Figure 8 shows the SERR for [+30/−30/90]$_s$ laminate with delaminations at the 90/90 interface, shown in Fig. 7(a).

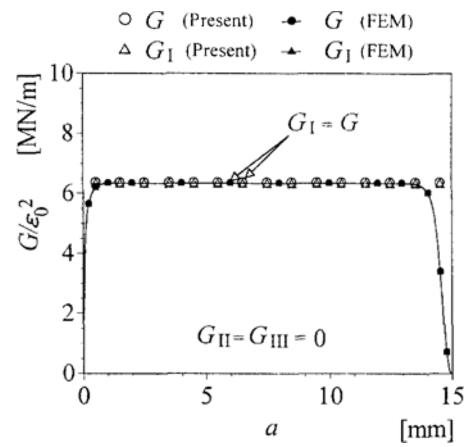


Fig. 8 Strain energy release rates for [30/−30/90]$_s$ laminate with free-edge delaminations at 90/90 interface.

The abscissa is the delamination crack length, and the ordinate is the SERR normalized by the square of the uniform axial strain ε_0. The total SERR G calculated by Eq. (22) coincides with Mode-Ⅰ SERR G_I obtained by the first equation of Eqs. (25). The Mode-Ⅱ and Mode-Ⅲ SERRs calculated from Eqs. (25) are zero. The Mode-Ⅰ component of SERR obtained by the present method is identical to the FEM with the virtual crack closure technique, when the delamination length is approximately more than the laminate thickness and when the delamination length is not too long. The SERR obtained from the FEM decreases as the delamination increases across the laminate width.

3.2 [+30/−30/90]$_s$ laminate with delaminations at the +30/−30 interfaces

Figure 9 shows the individual components of SERR for [+30/−30/90]$_s$ laminate with

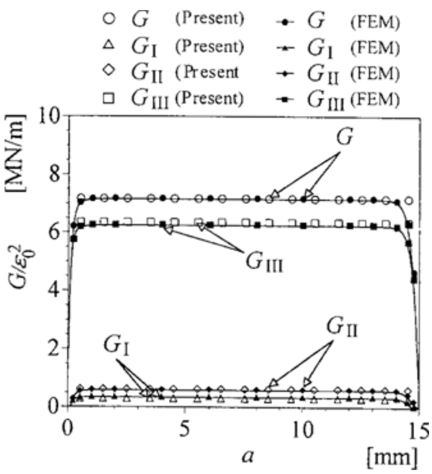


Fig. 9 Strain energy release rates for $[30/-30/90]_s$ laminate with free-edge delaminations at 30/ −30 interfaces.

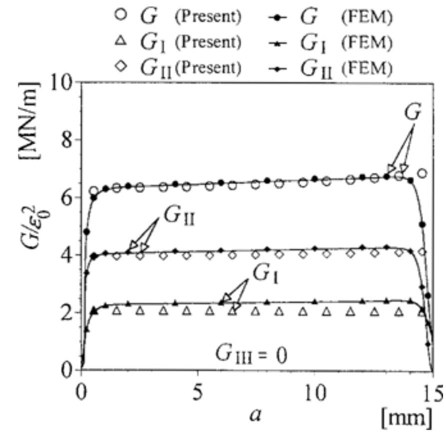


Fig. 10 Strain energy release rates for $[30/-30/90]_s$ laminate with free-edge delaminations at −30/90 interface.

delaminations at the +30/−30 interfaces, shown in Fig. 7(a).

The Mode-Ⅰ, Mode-Ⅱ, and Mode-Ⅲ components of SERR calculated by Eqs. (25) are in well agreement with the FEM results, when the delamination length is approximately more than the laminate thickness and when the delamination length is not too long. The mode components of SERR obtained from the FEM decrease as the delamination almost increases across the laminate width. The total SERR calculated by Eq. (22) coincides with the FEM result when the delamination length is neither too short nor too long.

3.3 $[+30/-30/90]_s$ laminate with asymmetric delaminations at the −30/90 interface

Figure 10 shows the individual components of SEER for $[+30/-30/90]_s$ laminate with delaminations asymmetric located with respect to the laminate midplane at the −30/90 interface, shown in Fig. 7(b).

The Mode-Ⅰ, Mode-Ⅱ, and Mode-Ⅲ components of SERR calculated by Eqs. (25) are in well agreement with the FEM results, when the delamination length is not too long. The mode components of SERR obtained from the FEM decrease as the delamination increases across the laminate width. The total SERR calculated by Eq. (22) coincides with the FEM when the delamination length is neither too short nor too long. For the case(3), however, the increase of the energy release rate is clearly recognized as the delamination grows. The results of the energy release rate analysis suggest that the behavior of the laminate depends on the position of the delamination interface.

3.4 Discussion

Wang(1984) showed that the SERR increases to a maximum value with increasing delamination length, beyond which it remains unchanged. He defined the delamination length at which this transition occurs as a characteristic delamination length. When predicting the onset and growth of free-edge delaminations, it is becoming generally accepted to determine the SERR for a delamination of the characteristic length or greater, and then to compare the calculated SERR with the critical value. The present method provides a good estimation of the individual components of SERR for the delamination of the characteristic length or greater.

The present method does not need the near-field information such as the displacement field or the distribution of the interlaminar stresses. Hence this method is not concerned with the oscillatory singularity. And, the analysis provides closed-form expressions for the Mode-Ⅰ, Mode-Ⅱ, and Mode-Ⅲ components of SERR. The simple nature of the method makes it suitable for preliminary design analysis for the delaminations of composite laminates.

4. Conclusions

Based on the previous sections, the following conclusions can be drawn :

(1) A simple method for determining the mode components of the strain energy release rate (SERR) of free-edge delaminations in composite laminates was developed. The method is based on a generalized quasi-three dimensional analysis, developed by modifying the classical laminated plate theory. The analysis provides closed-form expressions for the Mode-Ⅰ, Mode-Ⅱ, and Mode-Ⅲ components of SERR.

(2) The individual mode components of SERR obtained by the present method are in good agreement with the results of a finite element analysis using the virtual crack closure technique.

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